Harmonic Tidal Theory

An investigation of discrepancies between different implementations of the harmonic method

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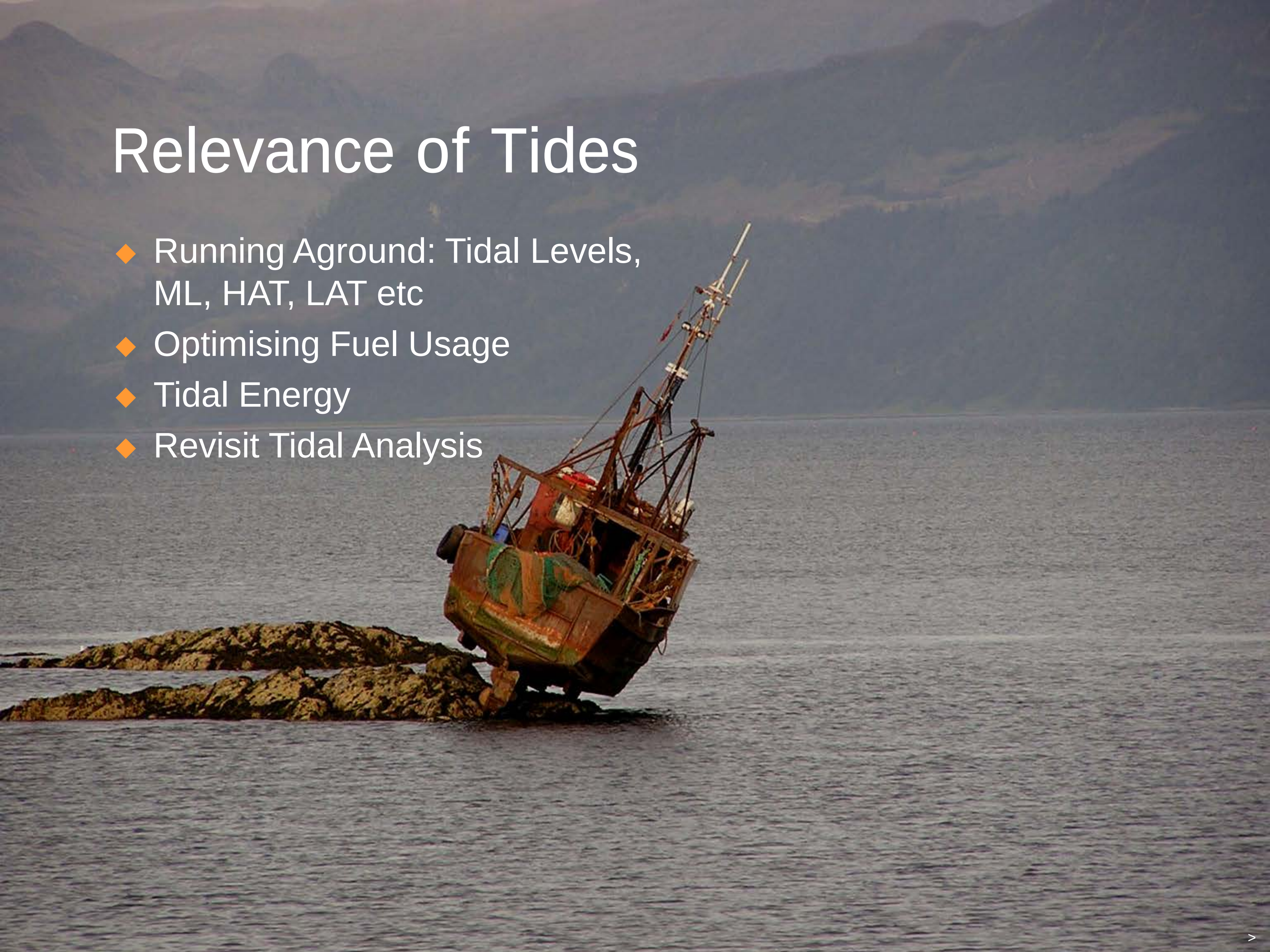
Outline

- Relevance of Tides
- History: Harmonic method
- Harmonic Listings
- Modulation - a variation of the harmonic method
- Compare two implementations
- Discussion of Discrepancies
- Summary and Conclusion

Relevance of Tides

- Running Aground: Tidal Levels, ML, HAT, LAT etc
- Optimising Fuel Usage
- Tidal Energy
- Revisit Tidal Analysis



History

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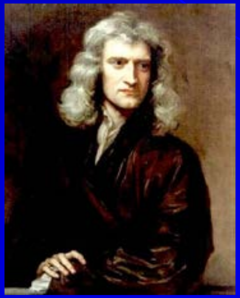
- 1687 Isaac Newton, Principia Mathematica
  - Superposition of tides
  - Diurnal inequality



- 1776 Laplace
  - Surpassed Newton's work
  - Eliminating the concept of bulges
  - Horizontal tidal force
  - "astres fictifs"

- 1822 Fourier
- 1872 Lord Kelvin's First Tidal Prediction Machine
- 1878 Hill's Lunar Orbit
- 1883 Sir G. H. Darwin harmonic tidal constituents.
- 1905 Brown's Lunar Orbit

Brown's - Mean Lunar Orbit 1905

- 655 terms for Lunar Longitude
- 300 terms for Lunar Latitude
- Accurate to within 0.1 seconds of arc
- Periodic terms all of known frequency

AT Doodson 1921

Re-calculated the tidal force using Brown's Lunar Orbit

Included all significant solar and lunar orbital effects

400 pure harmonic frequencies - none were modulated as Darwins were

Each with their own amplitude and phase

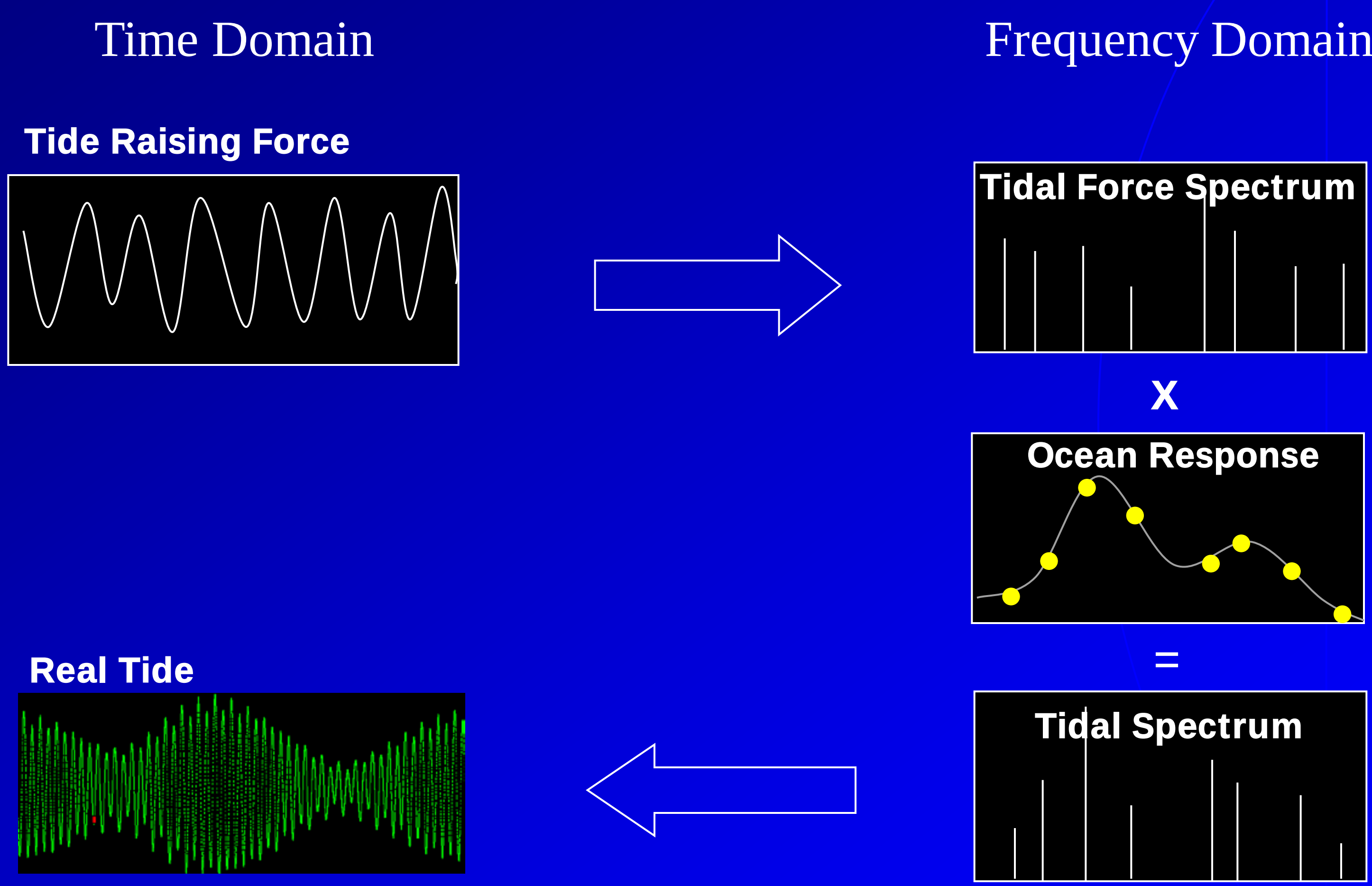
Ranges from 1 per 18.6 years to 3 times per day.

Harmonic Listings

- Pure Harmonic Listing
  - A.T. Doodson 1921
  - Cartwright & Tayler 1971
  - Buellesfield 1985
  - Tamura 1987
  - Xi Qin-wen 1987

- Listings which include modulation
  - Lord Darwin 1883
  - UKHO Admiralty Manual of Tides 1941 & numerous later publications
  - UKHO Simplified Method
  - IHO Tidal Committee 2003

Time & Frequency Domain



$$\text{Tide} = ML + \Sigma_{\text{all } i} \left(H_i . D_i \sin \left(w_i t + g_i + d_i\right)\right)$$

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Modulation

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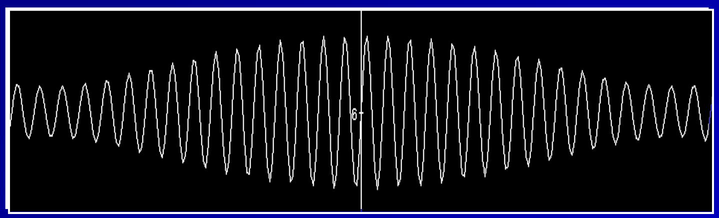
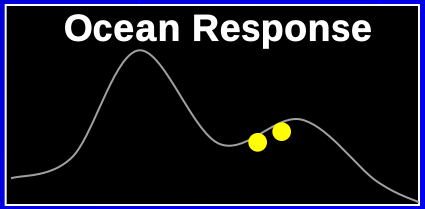
Time Domain

Frequency Domain



X



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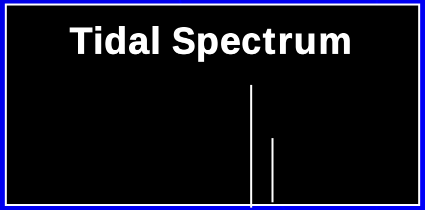
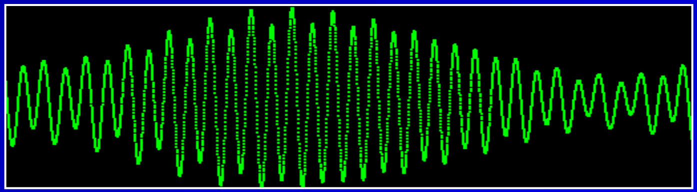


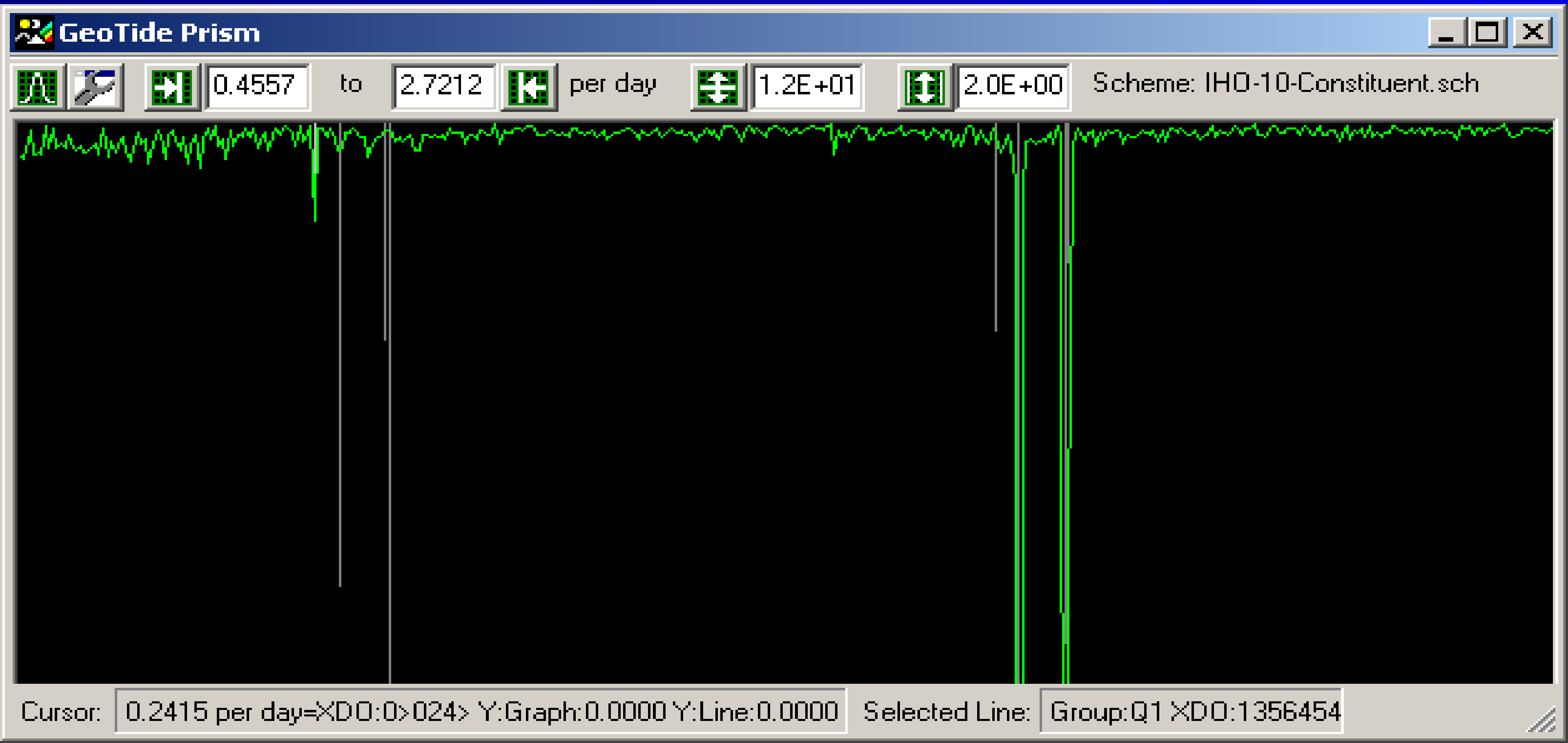
$\sin\left(\frac{w_{SD}}{2}\, t\right) \cdot \cos\left(\frac{w_f}{2}\, t\right)$

Real Tide



$\sin(w_1 t) + \sin(w_2 t)$

$$2 \sin\left(\frac{A + B}{2}\right) \cdot \cos\left(\frac{A - B}{2}\right) = \sin A + \sin B$$


GeoTide Prism
0.4557
to
2.7212
per day
1.2E+01
2.0E+00
Scheme: IHO-10-Constituent.sch
Cursor:
0.2415 per day=XDO:0>024> Y:Graph:0.0000 Y:Line:0.0000
Selected Line:
Group:Q1 XDO:1356454

Herne Bay

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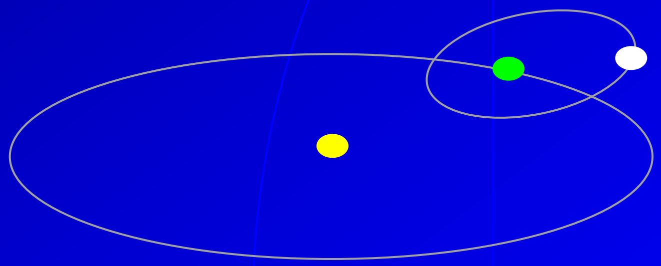
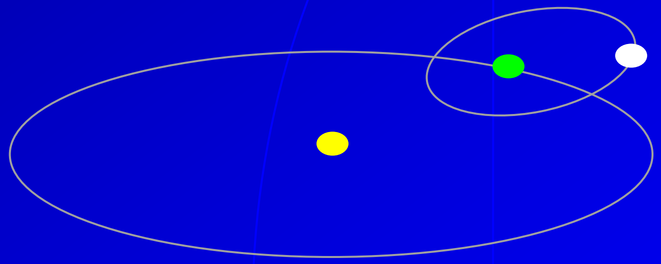
Lunar Orbit



- Lunar Orbital Complex
- Elliptical orbit ~ 5.5%
- Precession of line of the apsides: perigee & apogee
- Period 8.5 years
- Precession of Orbital Plane
- Period 18.6 years

Precession of Orbital Plane



- 18.6 years
- Changes the amplitude and phase of constituents
- "Nodal Modulation"
- Can be calculated with formulae or by adding constituents
- But both approaches should be equivalent
- How to compare

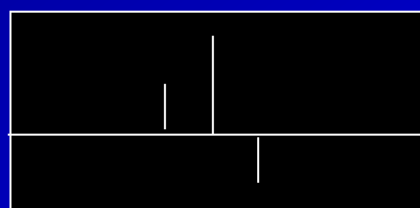
How to compare

1. Compare the published formulae with published constituent listings

2. Use tidal prediction software itself

Pure Harmonic Listing

$$h = (1/imax) \, . \sum I_i \, .\mathrm{Cos}\,(w_{SD} + j \, . \, N + \Phi_i)\, t \qquad \text{............(2)}$$



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0717555,0.00026'etcpot#: 95
0725565,0.00091'etcpot#: 98
0735455,0.00098'etcpot#: 105
0735555,0.01370'etcpot#: 106
0735657,0.00088'etcpot#: 107
0736554,0.00015'etcpot#: 110
0745547,0.00017'etcpot#: 119
0745565,0.00048'etcpot#: 121
```

MSf

Modulation Formulae

Time Domain

Modulation Formulae

$$h = f_{\text{of x}} . \text{Cos}\,(w_{SD} .t + u_{\text{of x}} .\pi / 180 + \Phi_i) \quad \text{............(3)}$$

Modulation includes both AM and FM

$$\mathbf{u_{MSf} = 2.14\ Sin\ N}$$

$$\mathbf{f_{MSf} = 1.0004 - 0.0373\ Cos\ N + 0.0002\ Cos\ 2N}$$

Comparison using Formulae

- Method
  - Enter previous two equations into a spreadsheet
  - Use normalised values for w
  - Calculate at 10,000 dates over 18 years
  - Calulate RMS error

	ETC Listing	Nodal Formulae	Normalis-ed RMS Error (%)
M2	2555355, 0.00047 2555457, 0.03386 2555555, 0.90812	u_{M2} = – 2.14 Sin N f_{M2} = 1.0004 – 0.0373 Cos N + 0.0002 Cos 2N [13]	0.01
M2	2555355, 0.00047 2555457, 0.03386 2555555, 0.90812	u_{M2} = – 2.14 Sin N f_{M2} = 1.0007 – 0.0373 Cos N + 0.0002 Cos 2N [19]	0.02
O1	1455356, 0.00218 1455454, 0.07105 1455554, 0.37689	u_{O1} = 10.80 Sin N – 1.34 Sin 2N + 0.19 Sin 3N f_{O1} = 1.0089 + 0.1871 Cos N – 0.0147 Cos 2N + 0.0014 Cos 3N [13]	0.03
O1	1455356, 0.00218 1455454, 0.07105 1455554, 0.37689	u_{O1} = 10.80 Sin N – 1.34 Sin 2N + 0.19 Sin 3N f_{O1} = 1.0176 + 0.1871 Cos N – 0.0147 Cos 2N [19]	0.62
K1	1655454, 0.01050 1655556, 0.53007 1655656, 0.07182 1655754, 0.00154	u_{K1} = – 8.86 Sin N + 0.68 Sin 2N – 0.07 Sin 3N f_{K1} = 1.0060 + 0.1150 Cos N – 0.0088 cos 2N + 0.0006 cos 3N [13]/[19]	0.01

Tidal Constituent	ETC Listing	Nodal Formulae	Normalised RMS Error (%)
MSf	0735455,0.00098 0735555,0.01370 0735657,0.00088	u of MSf = –u of M2 and f of MSf = f of M2 [13]/[19]	5.0
MP1	1475454,0.00014 1475556,0.00491 1475654,0.00107	u and f same as M_2 [19]	15.5
MP1	1475454,0.00014 1475556,0.00491 1475654,0.00107	u and f same as O_1 [13]	22.2

>

Comparison using Tidal Prediction Software

- TotalTide
  - Uses UKHO IHO constituent harmonic list
  - Implicitly uses nodal modulation formulae - UKHO / IHO documents.
- GeoTide
  - Uses pure summation of harmonic list from Doodson
  - as published by The Earth Tides Commission at International Association of Geodesy

Method

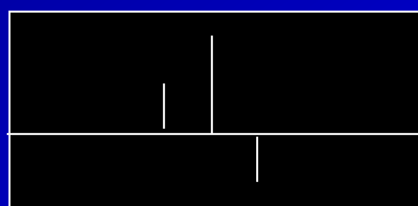
- Create artificial harmonic constants
- Set constituent under study to 100m
- At twenty random dates
- Predicted values were then analysed in a spreadsheethe
- Advantages
    - Independent of values of frequencies
    - Independent of software implementation

TotalTide Constituent	ETC Listing	Normalised RMS Error(%)
M2	2555355,2555457, 2555555	0.6
S2	2735555	0.0
K1	1655454, 1655556, 1655656, 1655754	0.1
O1	1455356,1455454, 1455554	0.5
Mnum	0636457,0636555, 0636657	3.4
Mm	0654457, 0654555, 0654657	0.3
MSf	0735455,0735555, 0735657	5.6
Mf	0755555, 0755655, 0755755, 0755857	0.8
2Q1	1257454,1257554	1.2
sig1	1275454,1275554	1.3
Q1	1356356,1356454, 1356554	0.8
rho1	1374454,1374554	0.8
MS1	1465444, 1465544	139.4
MP1	1475454, 1475556, 1475654	15.8
NO1	1556556,1556656	0.9
M1	1556556,1556656	46.5

Table 1. A Comparison of TotalTide and GeoTide on a constituent by constituent basis

MSf

- 5% Discrepancy
- ETC Listing: Its a Frequency Modulated tidal potential
- MSf is both a compound (M2-S2) and pure components
- In UKHO is modulated like -M2



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0725565,0.00091'etcpot#: 98
0735455,0.00098'etcpot#: 105
0735555,0.01370'etcpot#: 106
0735657,0.00088'etcpot#: 107
0736554,0.00015'etcpot#: 110
0745547,0.00017'etcpot#: 119
0745565,0.00048'etcpot#: 121
```

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	x
Mm		0.544 375	0 654 555	Z AZY ZZZ	y
MSf		1.015 896	0 735 555	Z BXZ ZZZ	b
MSo		1.015 896	0 735 555	Z BXZ ZZZ	b
SM		1.015 896	0 735 555	Z BXZ ZZZ	x
Mf		1.098 033	0 755 555	Z BZZ ZZZ	y

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M1/NO1

- M1 46%
- NO1 0.9%
- Same frequency
- But IHO Listing has 3 M1s
- ETC
- NO1 is not small

```
1555557,0.00661'etcpot#:  508
1555655,0.00086'etcpot#:  510
1556454,0.00085'etcpot#:  511
1556556,0.02964'etcpot#:  513
1556656,0.00594'etcpot#:  515
1556754,0.00017'etcpot#:  516
1565554,0.00016'etcpot#:  521
1566546,0.00018'etcpot#:  523
```

M1C		14.492 052	1 555 555	A ZZZ ZZZ	y
M1		14.492 052	1 555 556	A ZZZ ZZA	y
M1		14.492 052	1 555 557	A ZZZ ZZB	y
NO1		14.496 694	1 556 556	A ZZA ZZA	x
M1A		14.496 694	1 556 556	A ZZA ZZA	y
M1		14.496 694	1 556 556	A ZZA ZZA	y
LP1		14.569 548	1 574 554	A ZBY ZZY	x
chi1	(χ1)	14.569 548	1 574 556	A ZBY ZZA	j

MS1:

- ETC Listing
- IHO Listing
- TotalTide very large modulation
- Fortunately very small ~mm

```
1455554,0.37689'etcpot#: 449
1456455,0.00016'etcpot#: 454
1456557,0.00108'etcpot#: 455
1456655,0.00014'etcpot#: 456
1457556,0.00243'etcpot#: 458
1457656,0.00040'etcpot#: 459
1465444,0.00012'etcpot#: 462
1465544,0.00115'etcpot#: 463
1473556,0.00021'etcpot#: 470
1474557,0.00021'etcpot#: 472
1475454,0.00014'etcpot#: 474
1475556,0.00491'etcpot#: 476
1475654,0.00107'etcpot#: 477
1485546,0.00033'etcpot#: 482
```

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
nuK1	(vK1)	13.471 515	1 374 554	A XBY ZZY	x
O1		13.943 036	1 455 554	A YZZ ZZY	y
MK1		13.943 036	1 455 554	A YZZ ZZY	x
MS1		13.984 104	1 465 557	A YAZ ZZB	x
MP1		14.025 173	1 475 555	A YBZ ZZZ	m
MP1		14.025 173	1 475 556	A YBZ ZZA	m

MP1

- 15% difference
- ETC Listing
- 5mm
- The sidebands are same phase
- ETC is actually referring to Tau1 which is of identical frequency.
- Tau1 is not listed in TT

```
1473556,0.00021'etcpot#: 470
1474557,0.00021'etcpot#: 472
1475454,0.00014'etcpot#: 474
1475556,0.00491'etcpot#: 476
1475654,0.00107'etcpot#: 477
1485546,0.00033'etcpot#: 482
1526566,0.00014'etcpot#: 489
1536456,0.00063'etcpot#: 493
1536556,0.00278'etcpot#: 494
```

MK1		13.943 036	1 455 554	A YZZ ZZY	x
MS1		13.984 104	1 465 557	A YAZ ZZB	x
MP1		14.025 173	1 475 555	A YBZ ZZZ	m
MP1		14.025 173	1 475 556	A YBZ ZZA	m
tau1	(τ1)	14.025 173	1 475 556	A YBZ ZZA	k
M1B		14.487 410	1 554 554	A ZZY ZZY	y
M1B		14.487 410	1 554 556	A ZZY ZZA	y

Results Summary: Major Constituents

M2

- Predictor Comparison to 0.6%
- Formulae comparison agrees to 0.01% [13] or 0.02% [19]

S2

- The agreement is <0.01% in both tests

K1

- Predictor Comparison to 0.1%
- Formulae comparison agrees to 0.01%

O1

- Predictor Comparison to 0.5%
- Formulae comparison agrees to 0.03%[13] or 0.62%[19]

Depending on which published formulae is used ref 13 or ref 19.
13 Admiralty Manual of Tides AT Doodson 1941
19 IHO Tidal Constituents Table Vth IHO Tidal Committee 2003

Results Summary: Minor Constituents

MP1

Also ETC refers to Tau1 which is of almost identical frequency.

Also it suffers from an inconsistency in published nodal modulation formulae.

M1

3 components with identical name M1 are listed by IHO and it is not clear to which TotalTide refers.

The ETC component tallies with NO1 of the same frequency and agrees to within 0.9%

MSf

Is used to refer both to the astronomical constituent and the compound constituent S2-M2. Since these have different nodal formulae this could well account for the discrepancy.

MS1

ETC refers to a pure components while MS1, we probably refers to a compound component of similar frequency. However, MS1 in TotalTide seems to have a very large level of Nodal Modulation

Conclusions

- **Two comparison of two different variations on the harmonic method was attempted.**
- **On M2, S2, K1, O1 agreement was very good.**
- **The majority of minor constituents tested showed acceptable level of agreement but 3 discrepancies were noted (>4%) on minor constituents, which were explained mainly by ID issues.**
- **MSf may have two different types of nodal modulation depending upon the type of tide. ~mm**
- **Some inconsistencies in the formulae were also found in the literature which should be further tracked down.**

END

Additional Notes

073555 (MSf)

It is said in the admiralty manual of tides that this is a shallow water compound tide which probably masks the astronomical component. But is this the case. Could be interpreted as pure or as M2-S2 273555-255555 amplitudes are 0.9 and 0.1 respectively. In tha case of compound tide it also requires the coefficient of non-linearity. Perhaps ETC's 0735555 would correspond better to SM as listed in IHO but not in TotalTide.

155655 (M1/NO1)

It is surprisaing that agreement is better with NO1 rather than M1. However M1 appears 3 times: one entry has only a different phase - so thats two difrerent frequencies. The third entry is the same as NO1 but varying in nodal modulation formulae. It is not clear which M1 is referred to. ETC's 155655 would correspond therefore to NO1

In US Coastguards book M1 is listed at wto frequencies but with three origins one of shich is a compound tide.

1465544 (MS1)

ETC's 1465544 is probably not MS1 which is listed with a very slightly different frequency. MS1 must therefore be a compound having different nodal modulation.

147555   (MP1)
ETC's 147555 is probably Tau rather than MP1 of the same frequency. Unfortunatly Tau is not listed in TotalTide but has an amplitude of 5mm. The comparison was actually made with MP1.

Doodson Notation

7 digit number e.g. 2544667

Constituent ID= $d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7$

Apart from first digit - 5 must be subtracted.

Phase = 90° x (d_7 - 5) i.e. sin or cosine terms

Frequency = $0.96d_1 + 0.036d_2 + 0.0027d_3 + 0.00031d_4 + 0.00015d_5 + 0.00000013d_6$

Daily Month Year 8 y 18y 26,000y

Lunar Precession of Apsides Lunar Precession of Plane Solar Precession of Obliquity